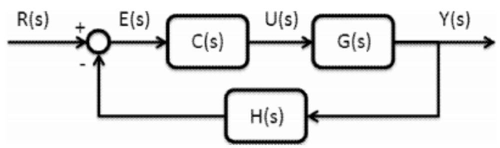
7. Generic Compensators

Electronic Control Systems

Prof. Rohan Munasinghe
Department of Electronic and Telecommunication Engineering
Faculty of Engineering
University of Moratuwa 10400

Problem of Root Locus Design

- Root locus design can locate poles only on the root locus. The method cannot change the root locus and locate poles at desired locations if those desired locations are not on the existing root locus.
- Generic compensators C(s) are used in such situations in order to change the existing root locus



- There are three generic compensators as
  - Phase lag compensator
  - Phase lead compensator
  - Notch filter

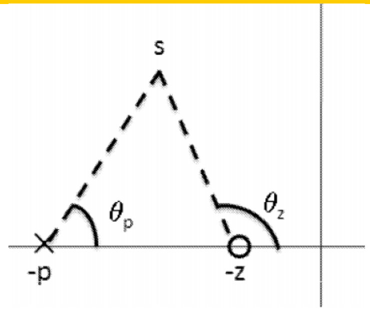
Phase Lead Compensator

- Transfer Function

$$C_{lead}(s) = \frac{s+z}{s+p}; p > z$$

- Phase contribution

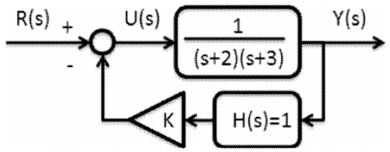
$$\theta_z - \theta_p > 0$$



- Example

Open loop transfer function of a system is $G(s) = \frac{1}{(s+2)(s+3)}$
Design a control system for the following specifications
1. 4% overshoot
2. 1% setting time of 1s

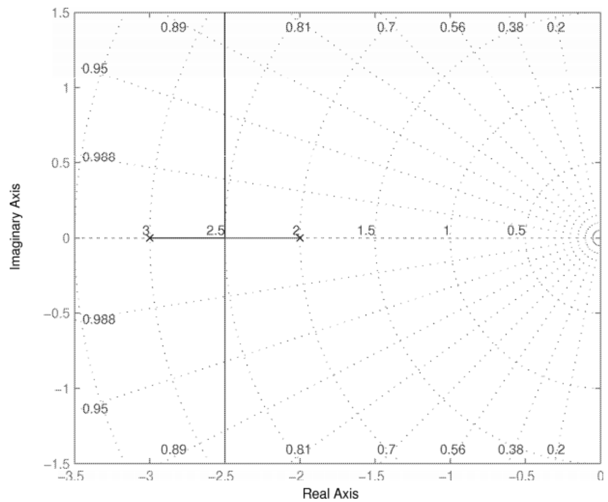
Phase Lead Design



Answer

The root locus of the feedback control system with a single feedback gain is

$$1 + K\frac{1}{(s+2)(s+3)} = 0$$



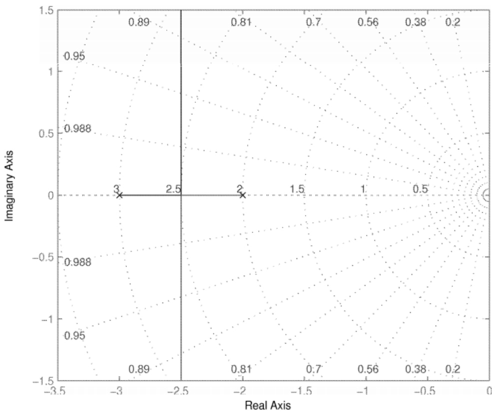
Phase Lead Design

$$\zeta = \sqrt{\frac{(\ln 0.04)^2}{\pi^2+(\ln 0.04)^2}} = 0.72 \qquad \omega_n = \frac{4.6}{1.0\times 0.72} = 6.39$$

- Desired poles for required specifications

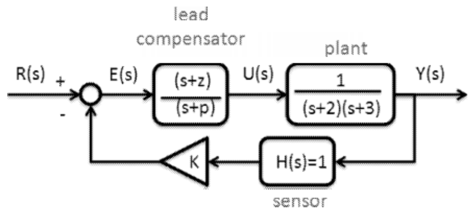
$$\gamma_1, \gamma_2 = \varsigma\omega_n \pm j\omega_n\sqrt{1-\varsigma^2}$$
$$= -4.6 \pm j4.43$$

- Poles are outside the root locus
- Gain tuning is not able to locate poles



Lead Compensator Design

- Root locus has to be bent to the left. Introduce a lead Compensator



- Characteristic Equation with the lead is

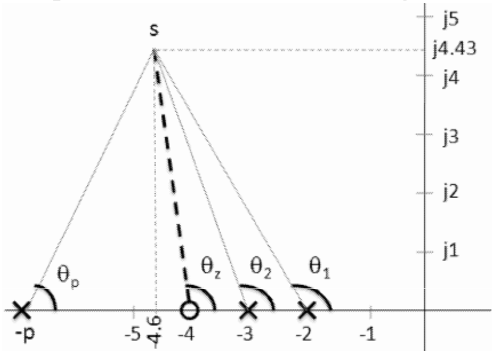
$$1 + K\frac{(s+z)}{(s+p)(s+2)(s+3)} = 0$$

- And, at $\gamma_1, \gamma_2 = -4.6 \pm j4.43$ phase condition

$$\angle\frac{(s+z)}{(s+p)(s+2)(s+3)} = \pm 180^0$$

Phase Lead Compensator Design

- Assume compensator zero=-4, and determine the pole



$$\theta_z - \theta_p - \theta_1 - \theta_2 = \pm 180(2k+1); k = 0, 1, 2...$$

$$\left[180^0 - \tan^{-1}\left(\frac{4.43}{0.6}\right)\right] - \theta_p$$
$$-\left[180^0 - \tan^{-1}\left(\frac{4.43}{1.6}\right)\right]$$
$$-\left[180^0 - \tan^{-1}\left(\frac{4.43}{2.6}\right)\right] = 180^0(2k+1)$$

Lead Compensator Design

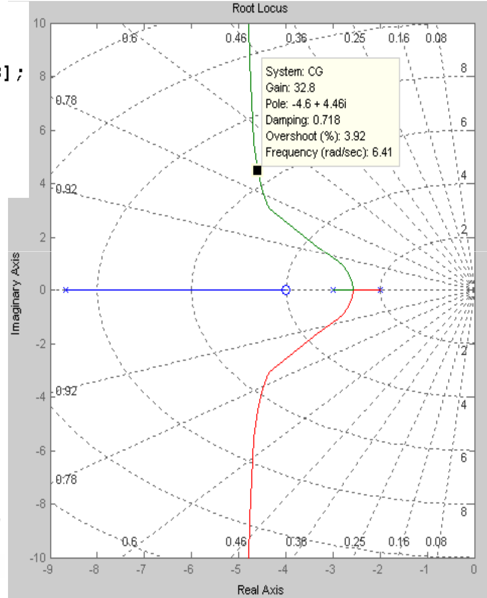
$$\begin{aligned}
(180^0 - 82.3^0) - \theta_p & \\
-(180^0 - 70.1^0) - (180^0 - 59.6^0) &= 180^0(2k+1) \\
97.7^0 - \theta_p - 109.9^0 - 120.4^0 &= 180^0(2k+1) \\
\\
-\theta_p - 132.6^0 &= 180^0(2k+1) \\
-\theta_p - 132.6^0 &= -180^0; k = 0 \\
\theta_p &= 47.4^0 \\
\tan^{-1}\left(\frac{4.43}{p-4.6}\right) &= 47.4^0 \\
\frac{4.43}{p-4.6} &= 1.09 \\
\frac{4.43 - 1.09\times 4.6}{1.09} &= p \\
8.66 &= p
\end{aligned}$$

Lead Compensator Design

$$\begin{aligned}
-\theta_p - 132.6^0 &= 180^0(2k+1)\\
-\theta_p - 132.6^0 &= -180^0; k=0\\
\theta_p &= 47.4^0\\
\tan^{-1}\left(\frac{4.43}{p-4.6}\right) &= 47.4^0\\
\frac{4.43}{p-4.6} &= 1.09\\
\frac{4.43-1.09\times 4.6}{1.09} &= p\\
8.66 &= p
\end{aligned}$$

Lead Compensator Design

```
% lead compensated root locus
num=[1 4];
den1=[1 8.66], den2=[1 2], den3=[1 3];
den=conv(den1,conv(den2,den3));
CG=tf(num,den);
rlocus(CG);
grid on;
```



- Closed Loop Tr Fn

$$\frac{Y(s)}{R(s)} = \frac{C(s)G(s)}{1+KC(s)G(s)}$$

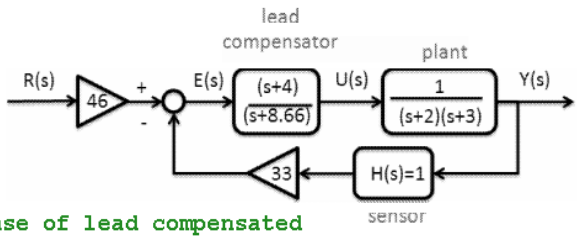
- DCG

$$\lim_{s\to 0}\frac{C(s)G(s)}{1+KC(s)G(s)}$$

$$\lim_{s\to 0}\frac{(s+4)}{(s+8.66)(s+2)(s+3)+33(s+4)}$$

$$0.0217$$

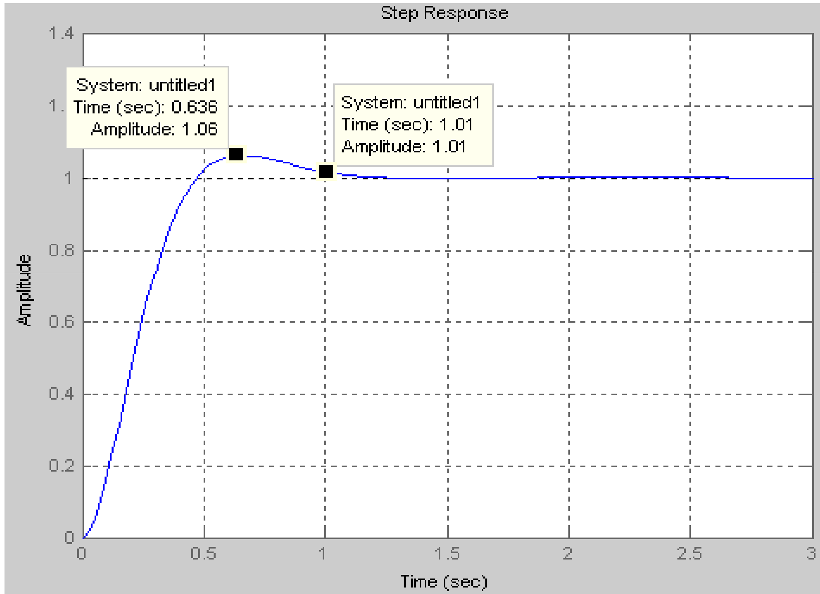
Lead Compensated System




```
% Step response of lead compensated
% system adjusted for DCG=1
dur=3; K=33;

num=[1 4];
den1=[1 8.66], den2=[1 2], den3=[1 3];
den=conv(den1,conv(den2,den3));
G=tf(num,den);

sys=feedback(G,K);

step(45.99*sys,dur);
grid on;
```

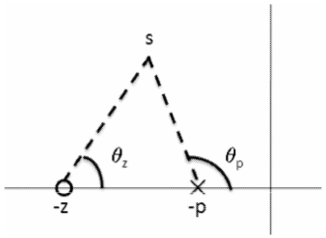
$$K = \frac{1}{0.217} = 45.99$$

Lead Compensator Design



Lag Compensator Design

- Lag transfer function $C_{lead}(s) = \frac{s+z}{s+p}; p < z$
- Phase contribution $\theta_z - \theta_p < 0$



- Lag compensator improves DCG

$$\lim_{s \to 0} \frac{G(s)}{1+KG(s)} = \frac{r}{1+Kr} \xrightarrow{\text{With Lag}} \lim_{s \to 0} \frac{C(s)G(s)}{1+KC(s)G(s)} = \frac{(\frac{z}{p})r}{1+K(\frac{z}{p})r}$$

$$r = \lim_{s \to 0} G(s) \qquad z >> p \quad \frac{(\frac{z}{p})r}{K(\frac{z}{p})r} = \frac{1}{K}$$

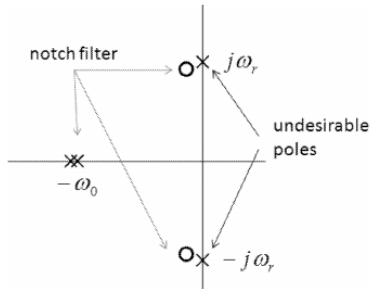
- Lag Compensator is rarely used as it reduces the phase margin in the feedback loop

Notch Filter Design

- Some plants show sustained oscillations even with a properly designed controller.
- NF can reduce the oscillatory response by way of pole-zero cancelation method- introducing two complex zeros very close to the oscillatory (pure imaginary) poles.

$$C_n(s) = \frac{s^2 + 2\varsigma\omega_0 s + \omega_0^2 + \omega_r^2}{(s+\omega_0)^2}$$

$$C_n(s) = \frac{(s+\varsigma\omega_0 + j\omega_r)(s+\varsigma\omega_0 - j\omega_r)}{(s+\omega_0)^2}$$



Notch Filter Design

Example

- Lead compensated feedback plant (previous work) is

$$\frac{Y(s)}{R(s)} = \frac{C(s)G(s)}{1+KC(s)G(s)} = \frac{46s+184}{s^3+13.66s^2+82.3s+184}$$

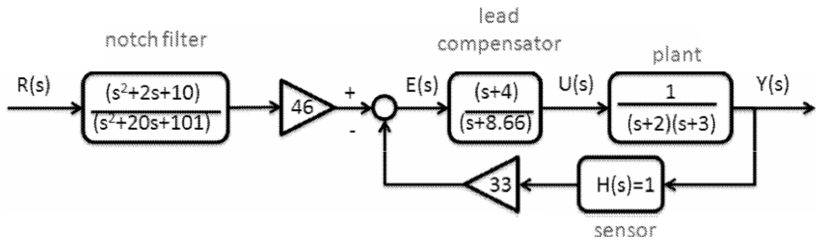
- Let's introduce a pair of oscillatory poles

$$\frac{Y(s)}{R(s)} = \frac{46s+184}{(s^3+13.66s^2+82.3s+184)(s^2+100)}$$

- NF poles and zeros $s = -1 \pm j10$, and two poles as $s = -10$
- NF Tr Fn $C_{ns} = \frac{(s+(1-j10))(s+(1+j10))}{(s+10)(s+10)} = \frac{s^2+2s+101}{s^2+20s+100}$

Notch Filter Design

- Notch filter can be inserted as a cascaded block with the existing feedback control system



Notch Filter Design

```
dur=3;   % simulation duration
K=33;    % feedback gain
g=46;    % gain for DCG=1
wr=10;   % freq. of sustained oscillation

% Lead compensator
numL=[1 4]; denL=[1 8.66];
Cs=tf(numL,denL);

% Plant
numG=[1]; denG1=[1 2]; denG2=[1 3];
denG=conv(denG1,denG2);
Gs=tf(numG,denG);

% feedback control system
OL=Cs*Gs;
CL=feedback(OL,K);

```

Notch Filter Design

```
% Unmodelled sustained oscillation
Ps=tf([1],[1 0 wr^2]);

% System with oscillation
sys1=g*CL*Ps;
subplot(2,2,1); rlocus(sys1,0); axis([-11 1 -11 11]); grid on;
subplot(2,2,2); step(sys1,dur); grid on;

% Notch filter
numN1=[1 1+j*wr];numN2=[1 1-j*wr]; numN=conv(numN1,numN2);
denN=[1 2*wr wr^2];
N=tf(numN,denN)

% System with notch filter
sys2=N*sys1;
subplot(2,2,3); rlocus(sys2,0); axis([-11 1 -11 11]); grid on;
subplot(2,2,4); step(sys2,dur); grid on;
```